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Abstract

Black [2] has recently formulated a model for the pricing of options on futures contracts under the assumption that futures contracts are equivalent to forward contracts. Since futures and forward contracts do differ because of the effects of changing interest rates, the volatility which is implied in the value of the option may be changing over the life of the option. This paper investigates the impact of changing volatilities on the pricing of options on futures by comparing the pricing performance of the Black model under the two alternative assumptions of constant and changing implied volatility. The empirical results, using options on New York Stock Exchange futures contracts, provide motivation for the development of a theoretical pricing model based on variable interest rates.

Stochastic Interest Rates, Changing Volatility and the Pricing of Options on Stock Index Futures

The original Black-Scholes [4] stock option pricing model assumes that the interest rate is constant over the life of the option. Merton [11] relaxed this assumption by introducing a variable interest rate option pricing model in which the capital market perceives a continuous sample path for equilibrium stock and default-free discount bond prices so that the option price is a linear homogeneous monotonic function of the ratio of the price of the underlying stock to the price of a default-free discount bond. Merton [11] has shown the Black-Scholes model to be a special case of his model in which the instantaneous bond price variance and the covariance between the stock and the bond price are both zero.

Using the same assumptions as Black and Scholes, Black [2] has developed a formula for pricing call options on futures contracts under the assumption that futures contracts are equivalent to forward contracts. This model differs from the original Black-Scholes model only in that the current stock price is replaced by the present value of the futures price. This transformation comes from the intuition that an investment in a futures contract requires no commitment of funds (see also [1], [12]).

Recent research ([5], [8] and [15]), however, has demonstrated that a futures contract will differ from a forward contract because of the "marking-to-market" effect, which in turn is a function of changing interest rates. Empirically, as Figure 1 illustrates, the volatility

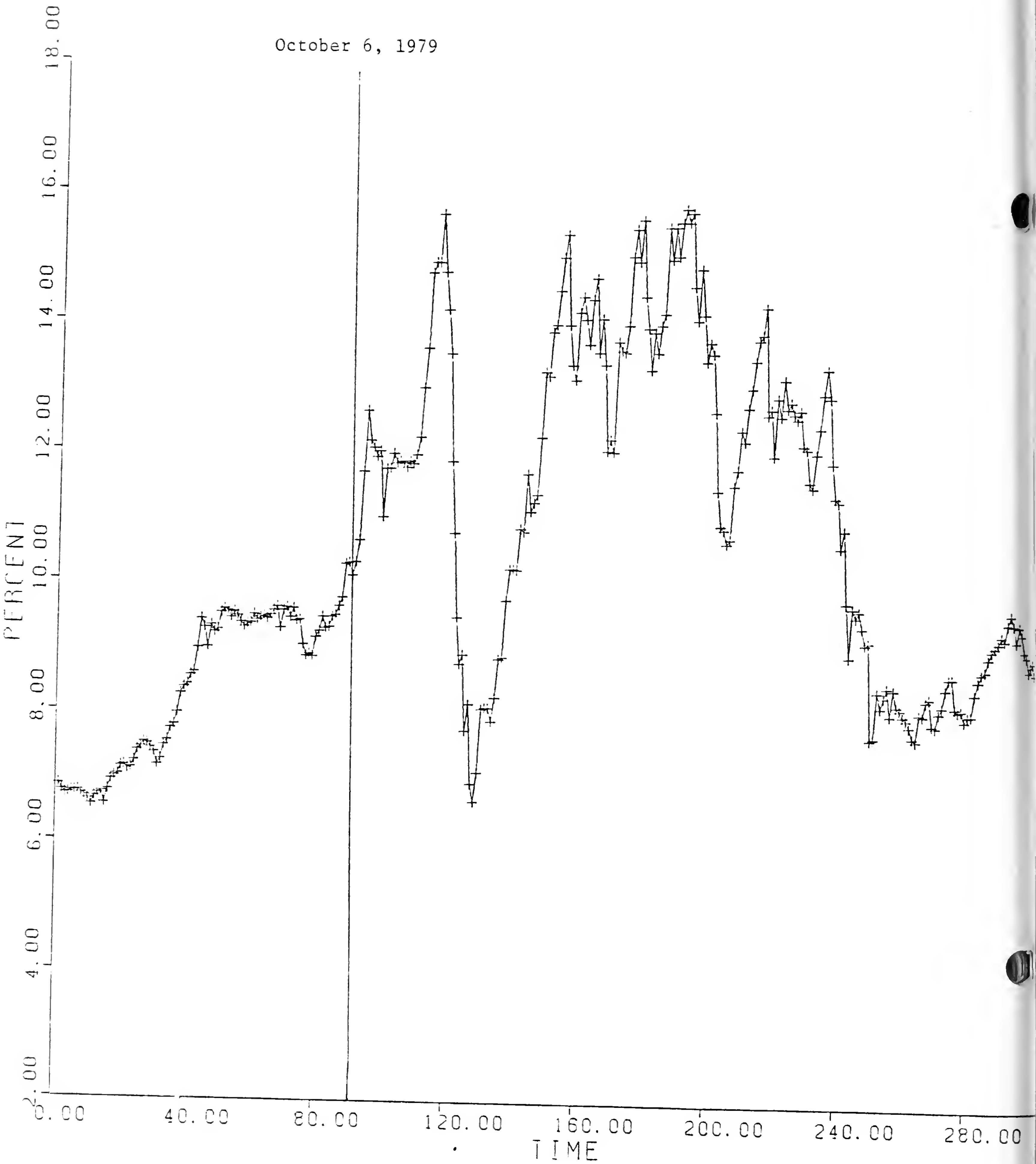

Figure 1. Weekly Averages of Six-Month Treasury Bill Yields for the Period: January, 1978 - September, 1983.

of short-term interest rates has changed dramatically over the past few years. This is due in part to a change in the Federal Reserve's policy (October, 1979) of monitoring monetary aggregates rather than interest rates. As such, it no longer seems reasonable to assume interest rates are constant and thus independent of futures price movements. Because the difference between futures and forward prices is affected by the covariability between futures and bond prices, it is apparent that the Black model may be misspecified because of this effect.

The purpose of this paper is to examine the efficiency of the Black model in the pricing of options on stock index futures and the impact of changing implied volatility, which is possibly due to a stochastic interest rate, on the pricing of such options. To facilitate this test, the Black model is examined under two alternative scenarios: constant and varying implied volatilities. The results indicate that the volatility which is implied in the value of an option on the NYSE stock index is changing over the life of the option. Recognizing this in the pricing process produces option prices significantly closer to their actual market values than is the case under the assumption of constant volatilities. These empirical results provide motivation for the development of a pricing model for options on futures along the lines of Merton's variable interest rate stock option model.

Section I describes the market for options on stock index futures and how the changing volatility can affect the pricing of such options. Section II describes the data base and methodology while Section III presents the results. A brief summary is contained in Section IV.

I. The Pricing of Options on Stock Index Futures

In February 1982, the Commodity Futures Trading Commission (CFTC) approved the trading of futures contracts on the Value Line Index at the Kansas City Board of Trade. This action was quickly followed by the introduction of futures contracts on the S and P 500 Index (Chicago Mercantile Exchange) and the NYSE Index (New York Futures Exchange) in April and May of 1982, respectively. These stock-index futures contracts differ from other physical commodity futures contracts because of their cash settlement procedure. These new futures contracts have maturity dates in the months of March, June, September and December. In 1983, the CFTC approved the trading of options on these new futures contracts. Options on the Value-Line futures are traded on the Kansas City Board of Trade, and options on the S and P 500 futures and the NYSE futures are traded on the Chicago Mercantile Exchange and the New York Futures Exchange, respectively. These options share the same maturity months as the corresponding futures contracts. A call (put) option on a futures contract conveys the right to go long (short) in a futures contract at a specific price (called exercise or striking price) during a specified time period.

A pricing model for call options on futures contracts was developed by Black [2] under the same assumptions as the original Black-Scholes model and is given in equation (1):

$$C = FN(d_1) - XN(d_2) \tag{1}$$

where:

C = the value of a call option on a futures contract

F = the present value of the futures price

X = the present value of the exercise price

= the price of a default-free discount bond which pays the exercise price on the expiration date

$d_1 = \ln(F/X)/\sigma_f\sqrt{t} + .5\sigma_f\sqrt{t}$

$d_2 = d_1 - \sigma_f\sqrt{t}$

σ_f^2 = instantaneous variance of percentage changes in futures prices

t = time to expiration of the option

$N(\cdot)$ = cumulative normal distribution

The impact that stochastic interest rates can have upon the pricing of options on futures can be examined within the context of Merton's variable interest rate stock option model. When interest rates are stochastic, the variance implied in the value of an option on a futures can be specified as:

$$\tau = \int_0^t S^2(t)dt \qquad (2)$$

where

$S^2 = \sigma_f^2 + \sigma_b^2 - 2\rho\sigma_f\sigma_b$

σ_b^2 = instantaneous variance of percentage changes in bond prices

ρ = correlation between changes in bond prices and futures prices

Note that the variance in Black's model is a special case of this specification in which $\sigma_b = \rho = 0$ and σ_f is constant.

The variable τ, called a "time" variable by Merton himself, is worthy of special notice, since all of the variables are directly observable in the market except the variance variable. Conceivably, the "time" variable can be split into two components: a pure time component and a volatility component. The former can be expressed as a fraction of a year and the latter as an annualized cumulative variate rate. In this manner, Black's model can be interpreted to be derived under the assumption that the volatility component is constant. In extending the Black-Scholes stock option model to include the possibility of stochastic interest rates, Merton [11] demonstrates that the correct maturity to form perfect hedge positions is the one which matches the maturity date of the options. (Note that in the Black-Scholes model, it does not matter whether hedgers borrow or lend long or short maturities since the model assumes a constant interest rate.) Given the empirical evidence of stochastic interest rates (see Figure 1) and thus non-constant instantaneous volatilities, the volatility component of τ is expected to be different for options on futures contracts of different maturities.[1] Therefore, by splitting the variable τ into two components, a time component and a volatility component, the impact of changing volatilities upon the pricing of options on futures can be examined. Empirically, the explanatory power of Black's model under the assumption of constant volatility can be examined by estimating the single implied standard deviation (ISD) which best fits groups of options of different maturities. On the other hand, the impact of changing volatility can be examined by estimating ISD's for options segregated by time to maturity. If the change in the underlying volatility is not important in the pricing of these options, then there should be no difference between the values determined by these two approaches.

II. The Data and Methodology

Daily closing call option and underlying futures price data for the NYSE stock index were gathered from the Wall Street Journal for the period: January 28, 1983-June 24, 1983 for options maturing in March, June, September and December of 1983.[2] January 28, 1983 marks the first trading day for options on NYSE index futures. This period provides a total of 2156 observations. Interest rates on United States Treasury Bills were gathered from the Wall Street Journal and updated daily. An average of the bid and asked discount rate for the Treasury bill having a maturity closest to the expiration date of the option was calculated and converted to an equivalent bond yield.

An important issue in previous empirical studies of option pricing models is the estimation of the expected volatility on the underlying security since, as Black and Scholes suggested, the usefulness of the model will depend on the market's ability to make an accurate estimate of this parameter. Previous approaches include, among others, calculating:

1) a historical estimate from ex-post price changes (Black and Scholes [3], and Galai [7])

2) using a weighted ISD (Schmallensee and Trippi [16], Latané and Rendleman [9], Chiras and Manaster [6], Patell and Wolfson [13] and MacBeth and Merville [10])

Schmallensee and Trippi and Patell and Wolfson calculate the implied standard deviation which would make the model price equal to the observed price for each option and then employ a simple equally-weighted arithmetic average of the ISD regardless of maturity. Latané and Rendleman weight each individual volatility estimate by the partial derivative of the model option price with respect to the ISD. Chiras and Manaster and MacBeth and Merville use a relative weighting technique following somewhat the same logic. While the discussion of all the pros and cons of these techniques is beyond the scope of this paper, prior research results tend to support the superiority of the ISD measure, regardless of the weighting scheme. This paper employs the technique introduced by Whaley[3] [18] which:

1) computes an implicit weighting scheme that yields an estimate of the volatility which minimizes the sum of squared errors and

2) calculates the implied volatility at time t-1 to circumvent the selection bias problem pointed out by Phillips and Smith [14]

The changing volatility issue is examined in the following manner. On each day during the period examined, Whaley's technique is used to compute ISD's in two ways:

1) all options with different exercise prices are lumped together, regardless of time to maturity, and a single ISD is computed

2) options are segregated according to time to maturity and a different ISD is computed for each time to maturity category

As such, two model prices are generated for each option in the sample where method 1) assumes volatilities are constant and thus prices all options with different times to maturity on a given day with the same ISD and method 2) recognizes that volatilities may change and prices options with different ISD's depending upon the time to maturity. We now examine the differences in these two pricing methods.

III. Empirical Results

For each futures, implied standard deviations are computed daily using a tolerance criterion of K = .0001 (see footnote 3). Figure 2 illustrates the daily behavior of the implied standard deviations for each of the four maturity cycles during the period examined. The first

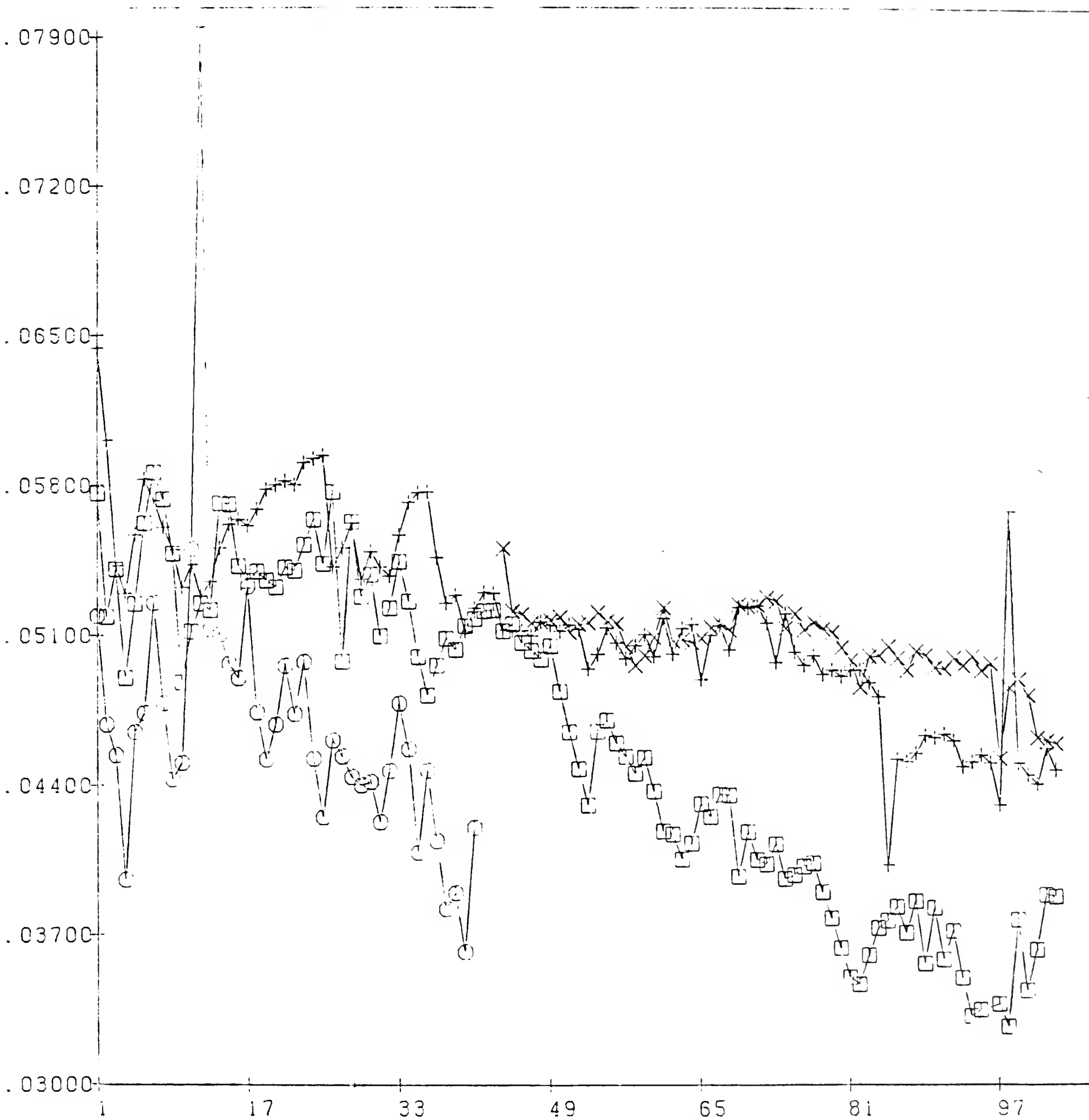

Figure 2. Implied Standard Deviations (ISD)
Across Time (measured in days)

Legend: O Mar 83, □ June 83, + Sept 83, X Dec 83 trading date (January 28, 1983) for options on NYSE index futures is designated as time 1. In general, the ISD estimates are declining over the life of each option. Equally importantly, the graph shows a wide divergence in the implied volatilities of the different contracts. In particular, the longer the time to maturity, the higher the variability implied in the option value. It appears that investors are uncertain about the true future variability of the market (e.g., NYSE futures) and thus translate alternative variability estimates into the prices of options written on futures of different maturities. In light of Merton's time variable, the declining ISD over the life of each option is consistent with the notion that as option approaches maturity, the market perceives less chance for bond price variability and hence covariability with futures prices.

Table I summarizes the differences between market prices and model prices of options calculated by the two aforementioned methods. Even though the models tend to misprice options in the same direction, differences between the two procedures are striking. When a single ISD is used, the average overpricing is 9.26 percent ($t = 12.1112$). However, even though the segregated (according to time to maturity) approach produces statistically significant mispricings ($t = 3.6054$), the magnitude of the average differential is only 1.14 percent.

We also examine the impacts of exercise price (X/F) and time to maturity (t) on differences between market prices and model prices. With the exception of in-the-money options ($X/F < .95$), prices based upon segregated maturities (differing volatilities) are significantly closer to actual market values than are prices based upon constant implied

Table I

Differences between Market Prices and Model Prices[a]

$$\text{Average \% difference with constant volatility} = \frac{B(\text{model}) - A(\text{actual})}{A(\text{actual})} \times 100$$

$$\text{Average \% difference with varying volatility} = \frac{M(\text{model}) - A(\text{actual})}{A(\text{actual})} \times 100$$

	(B-A)/A	(M-A)/A	(M-B)/A[c]	Number of Observations
All options	9.26% (12.1112)	1.14% (3.6054)	-7.99% (-9.7780)	2156
X/F < .90	-1.55 (-17.1255)	-1.58 (-18.8868)	-.03[b] (-.2434)	231
.90 ≤ X/F < .95	-.93 (-9.9891)	-1.22 (-17.3851)	-.29 (-2.4604)	516
.95 ≤ X/F < 1.05	13.12 (12.5890)	2.68 (7.1041)	-10.44 (-9.4167)	1398
X/F ≥ 1.05	31.42 (4.3147)	-.30[b] (-.0924)	-31.72 (-3.1525)	111
t < 3 months	24.80 (12.47564)	1.69[b] (1.9784)	-23.11 (-10.6745)	759
3 months ≤ t < 6 months	2.53 (7.5196)	1.21 (4.9733)	-1.32 (-3.1445)	743
t ≥ 6 months	-1.57 (-12.3579)	.42 (3.4036)	1.99 (11.2378)	654

[a]Numbers in parentheses are t values

[b]Not significantly different from zero at the 1% level

[c]Difference between (M-A)/A and (B-A)/A

volatilities. (Note, however, that for in-the-money options, the differences in mispricing between the two alternatives are only .03 percent and .29 percent when X/F < .90 and .90 $\leq$ X/F < .95, respectively.) Differences between market prices and model prices for different X/F values under the assumption of constant volatility and varying volatility are graphically displayed in Figures 3A and 3B, respectively. As the scales on these figures indicate, the constant volatility (3A) approach can produce large percentage deviations (several are in excess of 1.00 (100%)).

With regard to the time to maturity effect, differences between the two pricing formulations are also significant, most notably in options with maturities less than 3 months. The magnitude of mispricing under the assumption of constant volatility varies much more across the alternative times to maturity categories than does the one under the varying volatility assumption. For example, the average mispricings across maturities ranges from 24.80% to -1.57% for the constant volatility column. On the other hand, with varying volatilities, the mean mispricing levels are quite similar (1.69% to .42%). Since the ISD measured in the first column is an average of the maturity classes' ISD's, it is not surprising that the least amount of relative mispricing (statistically) occurs for intermediate term options (3 months $\leq$ t < 6 months) when all options are pooled to compute the ISD. From Figure 2 it can be seen that the intermediate term ISD most nearly approximates the overall ISD. Because the three-month ISD differs more from the other maturity ISD's, a pooled estimate will most seriously misprice these options. This overpricing effect is consistent with the failure

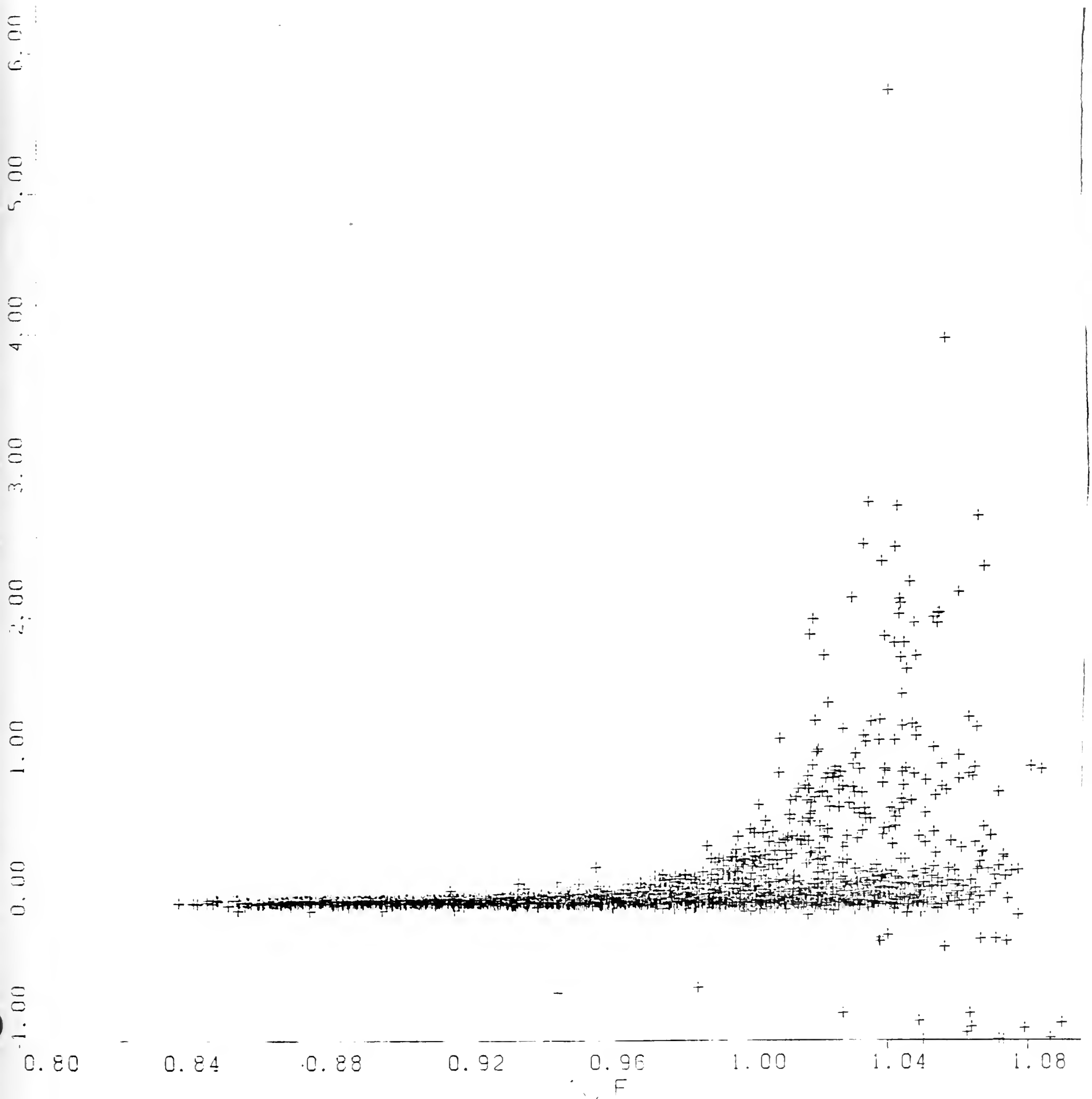

Figure 3A. Differences between Market Prices and Model Prices for Different X/F Values Under the Assumption of Constant Volatility

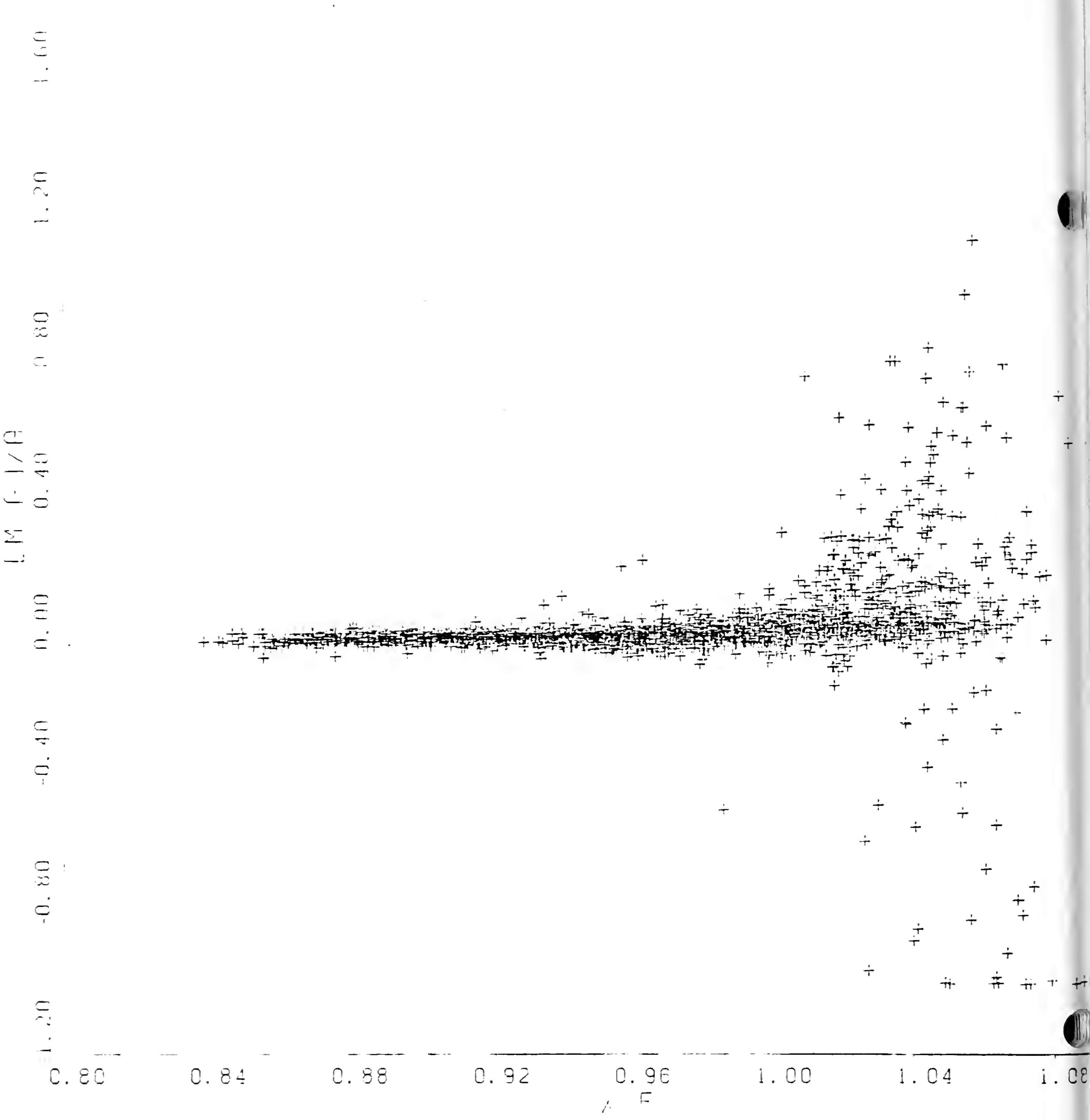

Figure 3B. Differences between Market Prices and Model Prices for Different X/F Values with Varying Volatility of the model to correctly measure a downward changing implied volatility. The mispricing versus time dimension (as measured by months to maturity) is displayed in Figures 4A and 4B under the two pricing formulations.

As a final test, all of the observations are pooled and the relationships between mispricing and X/F and time to maturity are examined and presented in Table II. Segregating according to maturity in order to capture the changing volatility reduces significantly the pricing bias associated with the exercise price/futures price ratio and for the sample as a whole eliminates any bias due to time to maturity.

IV. Summary

Interest rate variability has increased dramatically in the last few years. Theoretically, this gives rise to non-constant volatilities which are implied in the values of options traded on futures contracts. Empirical evidence of this non-stationarity can be obtained through an examination of the implied volatilities on option contracts of differing maturities. Correcting for this problem by pricing options with ISD's derived from segregating maturities provides a more accurate assessment of market values. These empirical results provide motivation for the development of a more precise option on futures pricing model--one which incorporates the impact of changing volatility on value estimation.

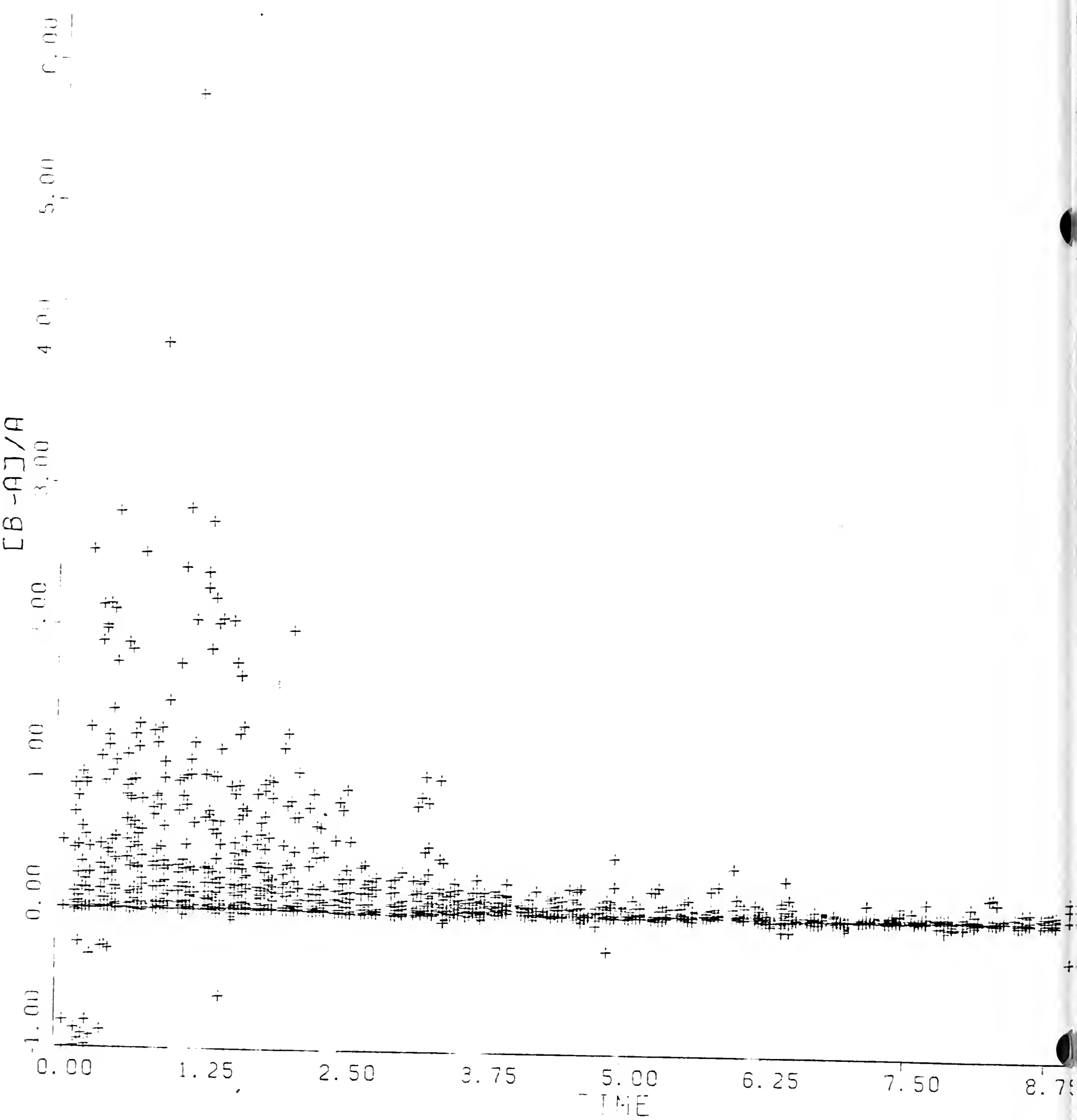

Figure 4A. Differences between Market Prices and Model Prices for Different Times to Maturity Under the Assumption of Constant Volatility. Time is measured as the number of months to maturity.

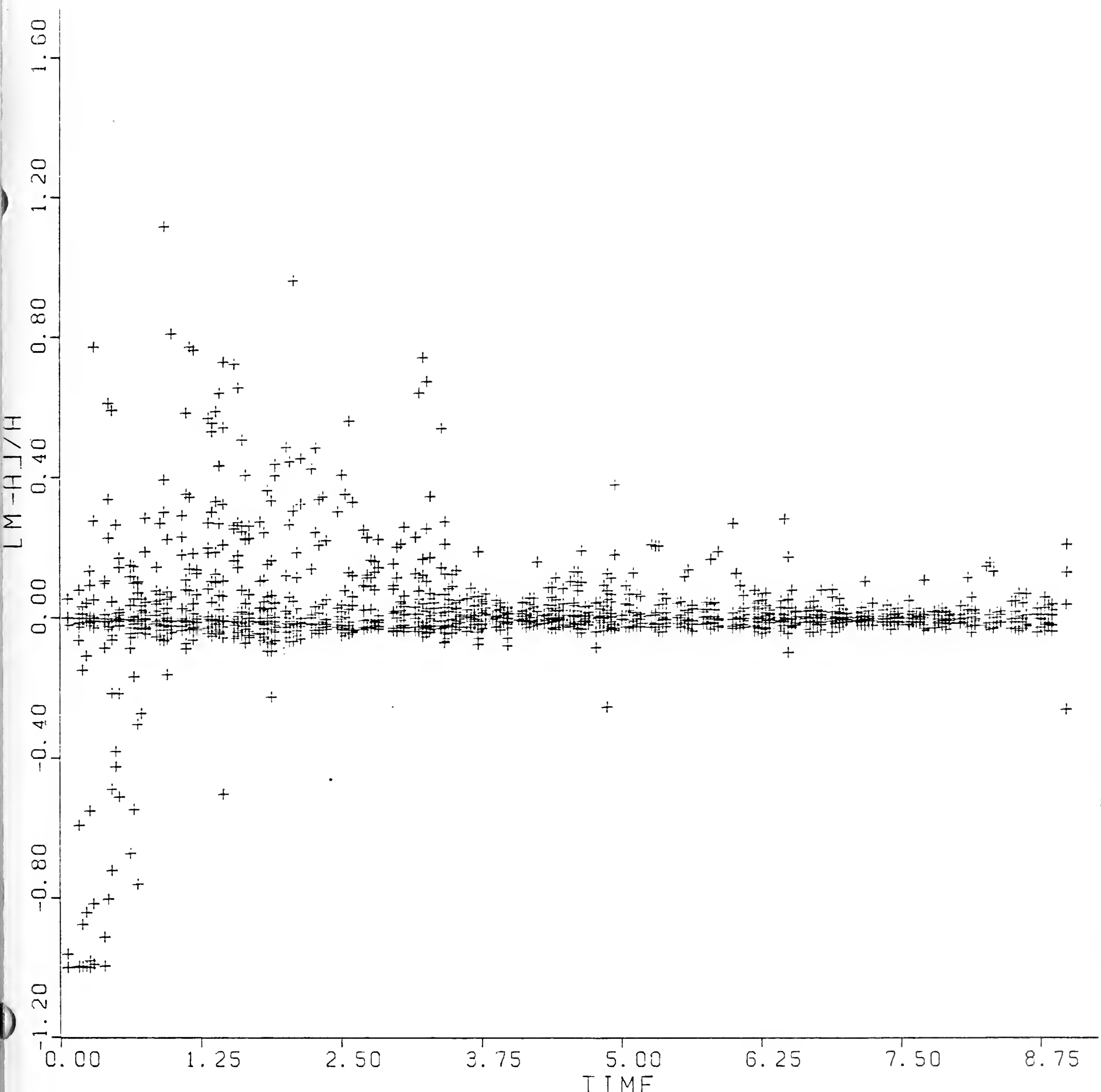

Figure 4B. Differences between Market Prices and Model Prices for Different Times to Maturity with Varying Volatility. Time is measured as the number of months to maturity.

Table II

Regression Results[a]

Test	α_0	α_1	$\bar{R}^2$	SSE[c]
1. $(B-A)/A = \alpha_0 + \alpha_1(X/F)$	-2.0643 (-15.4532)	2.2191 (16.1593)	.1077	.1093
$(M-A)/A = \alpha_0 + \alpha_1(X/F)$	-.4475 (-7.6677)	.47241 (7.8737)	.0275	.0209
2. $(B-A)/A = \alpha_0 + \alpha_1 t$	.2737 (19.7077)	-.0439 (-15.3925)	.0980	.1105
$(M-A)/A = \alpha_0 + \alpha_1 t$	.0177[b] (1.9066)	-.0001[b] (-.0570)	-.0005	.0215

[a] numbers in parenthesis are t values

[b] not significant at the 1% level

[c] sum of squared errors

Footnotes

[1]This effect can be seen through an examination of the time variable in the Merton model. Conceivably as the bond approaches maturity, its price converges to the face value; thus, its variance rate and the covariance term approach zero for the same reason. Thus, even though the futures price variance were constant through time, the instantaneous variance of the price ratio need not be constant over time. Another reason for possible differences in volatility across maturities, as noted by Pattell and Wolfson [13] and cited by Whaley [18], is the effect of anticipated information arrival. For instance, consider the information effect on options for different maturities when the information set is limited to news about inflation. One would not be surprised to see a higher implied standard deviation in options which expire after the anticipated announcement date of news about inflation. As there might be many such anticipated information arrivals, a variety of frequently changing implied volatilities might be anticipated with the characteristic being a higher implied volatility for longer-maturity options (see [13]). This effect would be stronger the greater the uncertainty about inflation (see figure 1).

[2]We also obtained daily closing call option and underlying futures price data for the S & P 500 index from the Wall Street Journal for the same time period. The results for the S & P 500 options do not deviate significantly from those for the NYSE options and are not reported here to conserve space. They are available from the authors upon request.

[3]Whaley's procedure to estimate the ISD can be summarized in the following manner. First, options written on the same security (at a given point in time) can be expressed as:

$$C_j = \hat{C}(\sigma)_j + \varepsilon_j \qquad \text{(a)}$$

where: C_j = the market price of option j

$\hat{C}(\sigma)_j$ = the model price

ε_j = the residual

An estimate of σ is determined by minimizing the sum of the squared observed residuals, $\hat{\varepsilon}_j$. An iterative (non-linear) technique is used to minimize the sum of the squared residuals by first obtaining an initial estimate of σ by using a Taylor series approximation:

$$C_j = \hat{C}(\sigma_0)_j + \partial\hat{C}_j/\partial\sigma_0(\sigma-\sigma_0) + \ldots + \text{higher order terms} + \varepsilon_j \qquad \text{(b)}$$

where σ_0 = initial value of volatility

σ = true volatility

Assuming that the higher order terms are trivial, (b) can be written as:

$$C_j - \hat{C}(\sigma_0)_j = \partial\hat{C}_j/\partial\sigma_0(\sigma-\sigma_0) + \varepsilon_j \qquad \text{(c)}$$

An estimate of the volatility, $\hat{\sigma}$, is found by applying OLS repeatedly until the estimate satisfies an accepted tolerance K:

$$|(\hat{\sigma}-\sigma_0)/\sigma_0| < K$$

where K = .0001. We use the same criterion as Whaley's in this paper.

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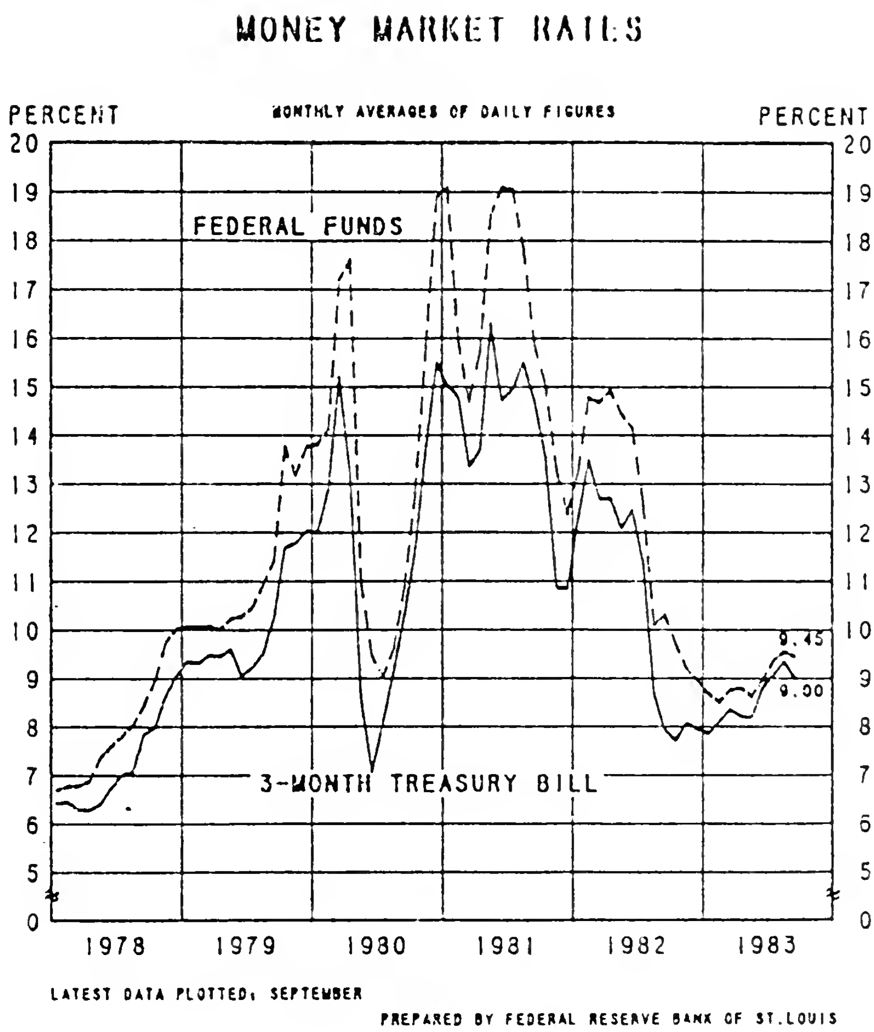

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